EINN* 2009

Hadron EM Form Factors: Theoretical Review

Lorcé Cédric
Mainz University
Germany

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

***Milos Conference Center George Eliopoulos, Milos Island, Greece (sept 28 – oct 2)**

Outline

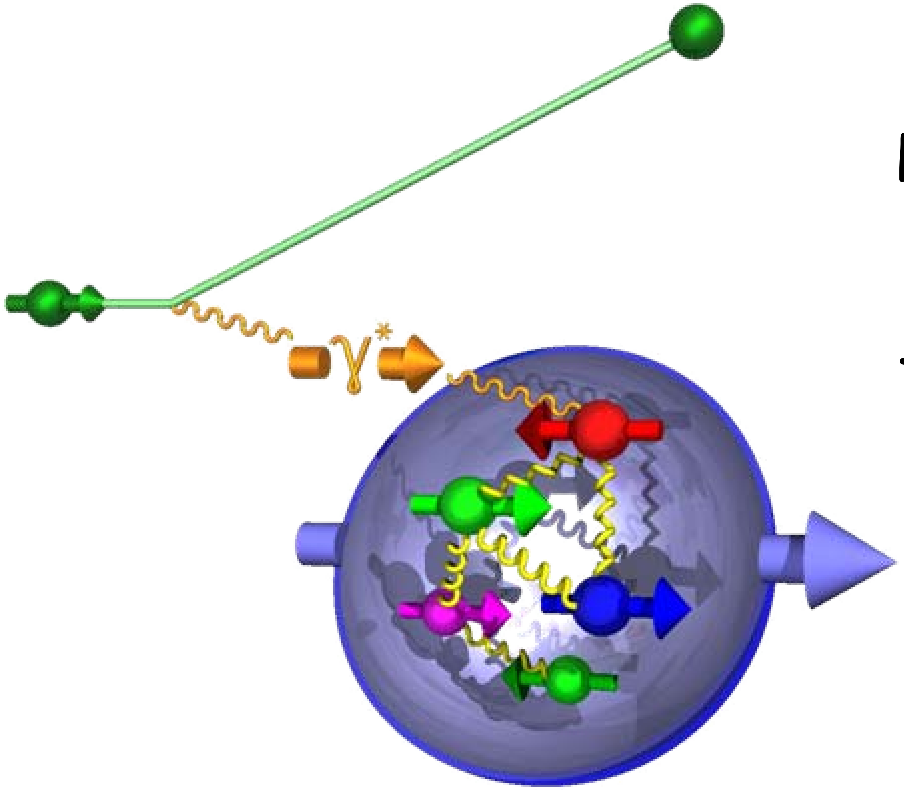
- Motivation & Definition of FFs
- Nucleon FFs : experimental status
- Theoretical approaches
  - χPT, Lattice QCD, GPDs, pQCD
- Radiative corrections
- Physical Interpretation of FFs

NB: time-like FFs *cf.* talk by S. Pacetti

→ Recent reviews: [Hyde-Wright *et al.* (04)]
[Arrington *et al.* (06)]
[Perdrisat *et al.* (06)]

Motivation

Understanding the hadron internal structure



Probe: γ

Targets: N, Δ, π, …

Definition of Form Factors

- $\alpha_{\mathrm{EM}} \sim 1/137$ → single-photon exchange (Born approximation)
- Hadronic current (elastic scattering on spin-1/2)
  - Relativistic covariance
  - Current conservation

$$q = p' - p, \quad q^2 = -Q^2$$

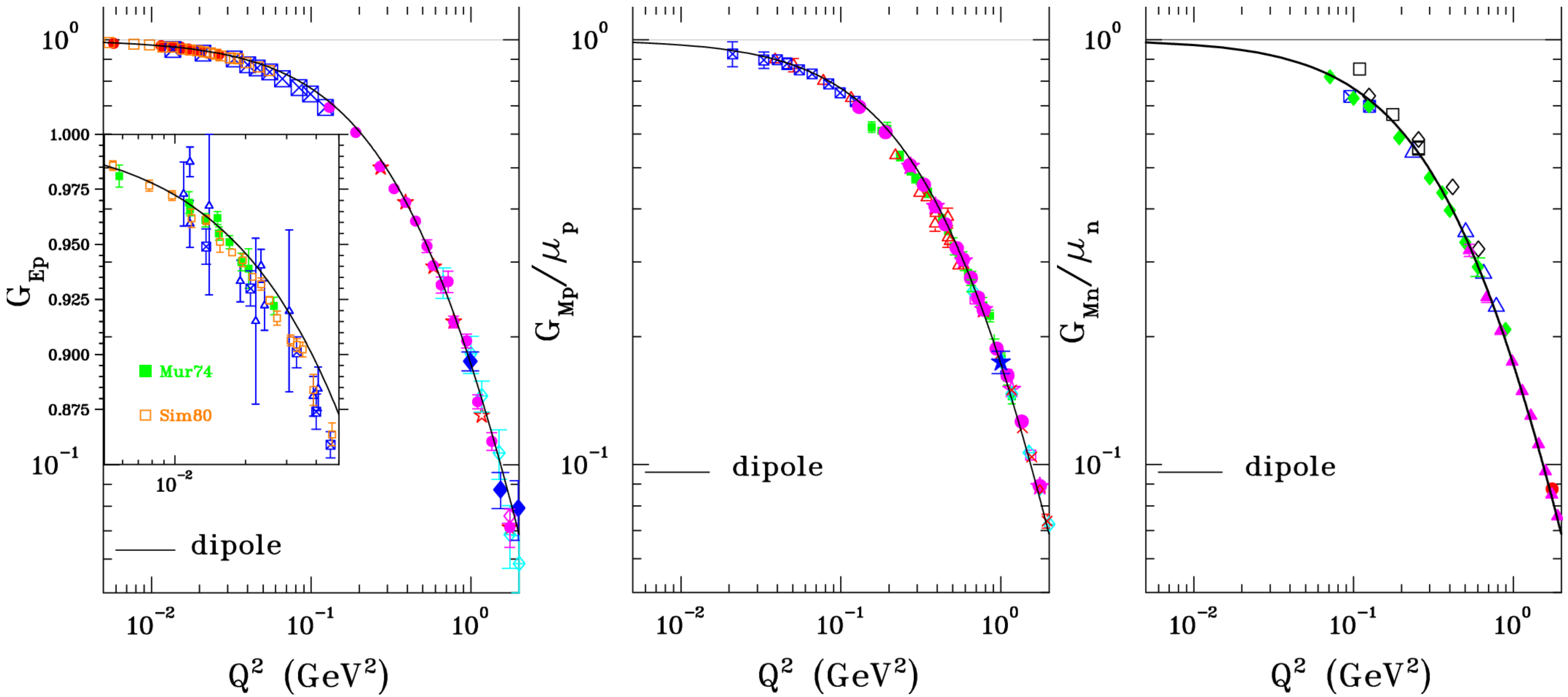
$$\langle p', \lambda' | J^{\mu}_{\mathrm{EM}} | p, \lambda \rangle = \bar{u}(p', \lambda') \left[\gamma^{\mu} F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_{\nu}}{2M} F_2(Q^2) \right] u(p, \lambda)$$

- Sachs Form Factors

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Dipole Approximation

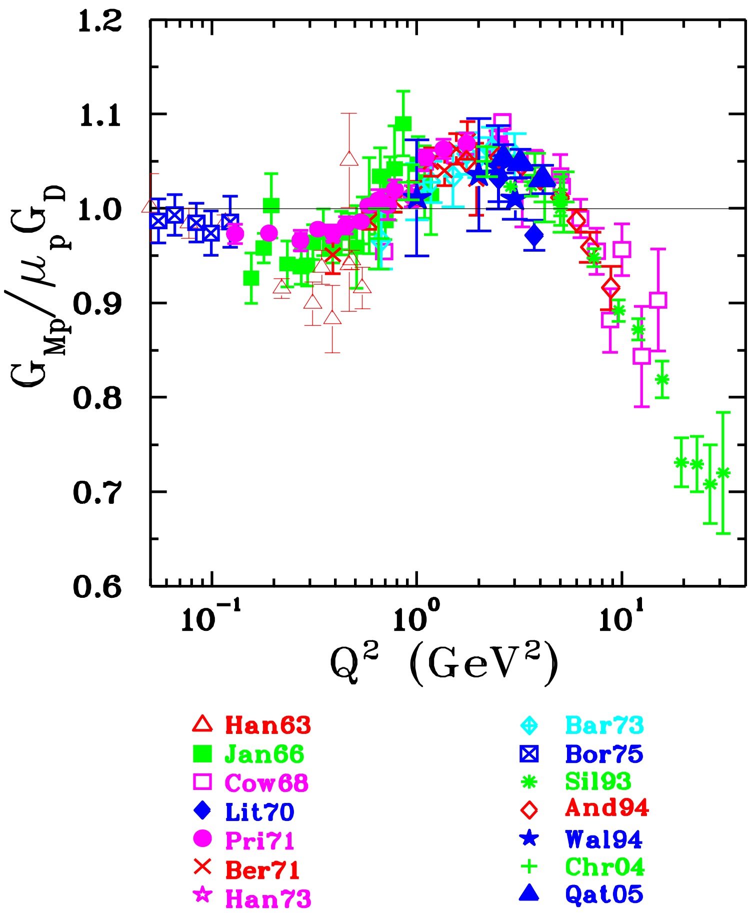


[Perdrisat *et al.* (06)]

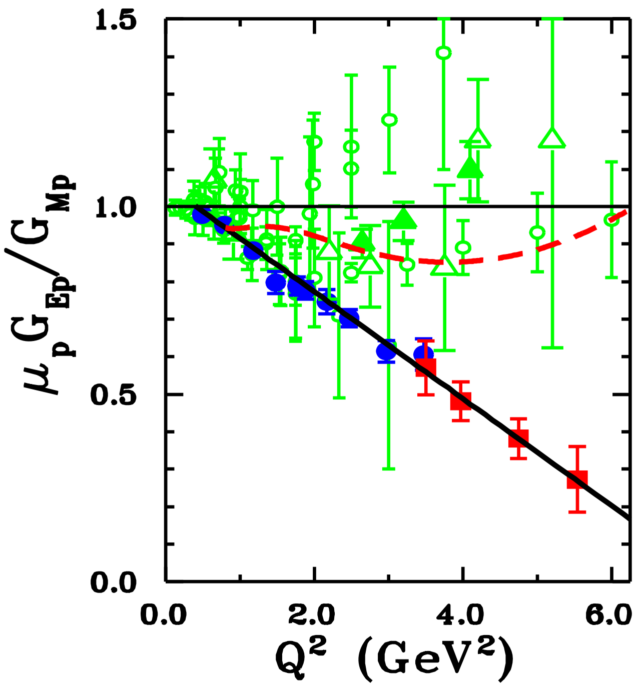
$$G_D = \frac{1}{1 + Q^2/\Lambda_D^2}, \qquad \Lambda_D^2 = 0.71 \text{ GeV}^2$$

Proton FFs: status

talk by F.-X. Girod



new MAMI/A1 data up to $Q^2 \approx 0.7$ GeV^2



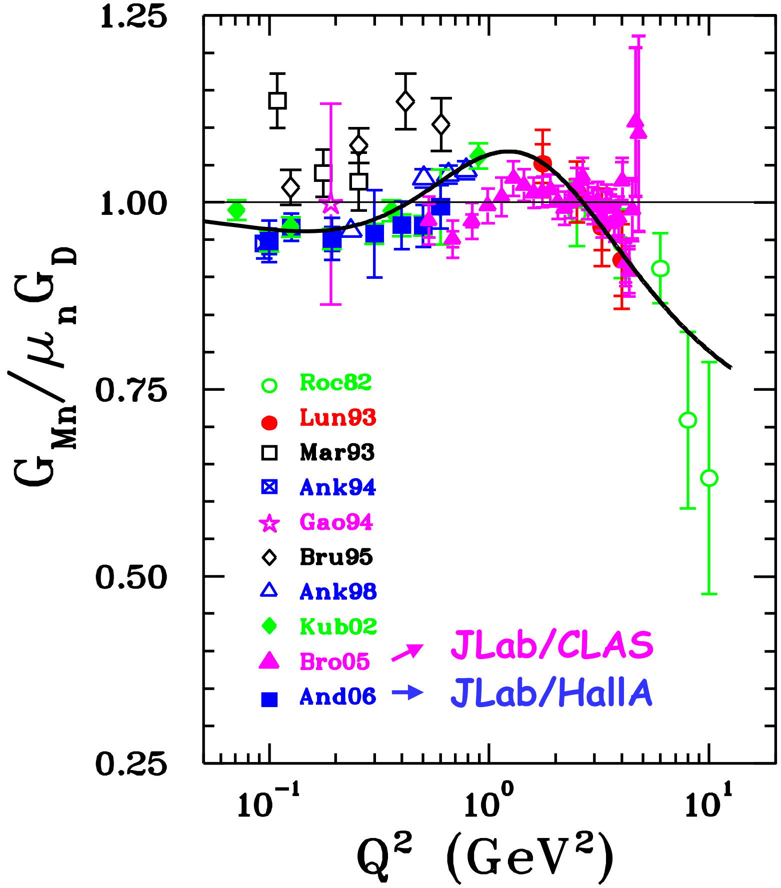
green : Rosenbluth data (SLAC, JLab)

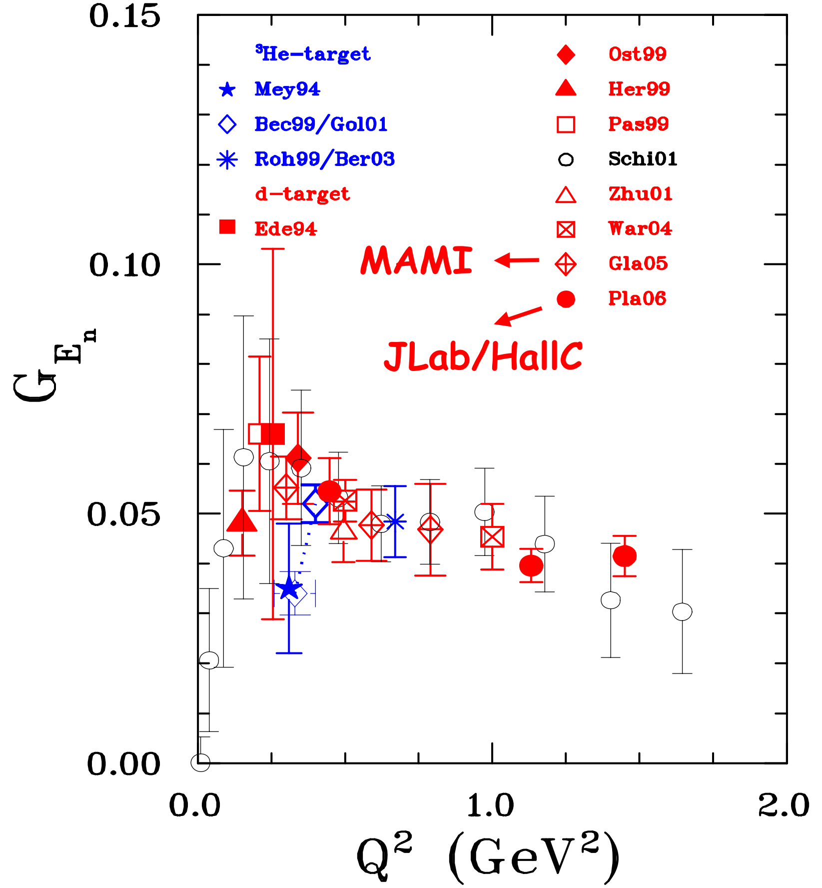
Pun05, Gay02 } JLab/HallA recoil polarization data

new JLab/HallC recoil pol. exp. (spring 2008) : extension up to $Q^2 \approx 8.5$ GeV^2

Neutron FFs: status

talk by F.-X. Girod

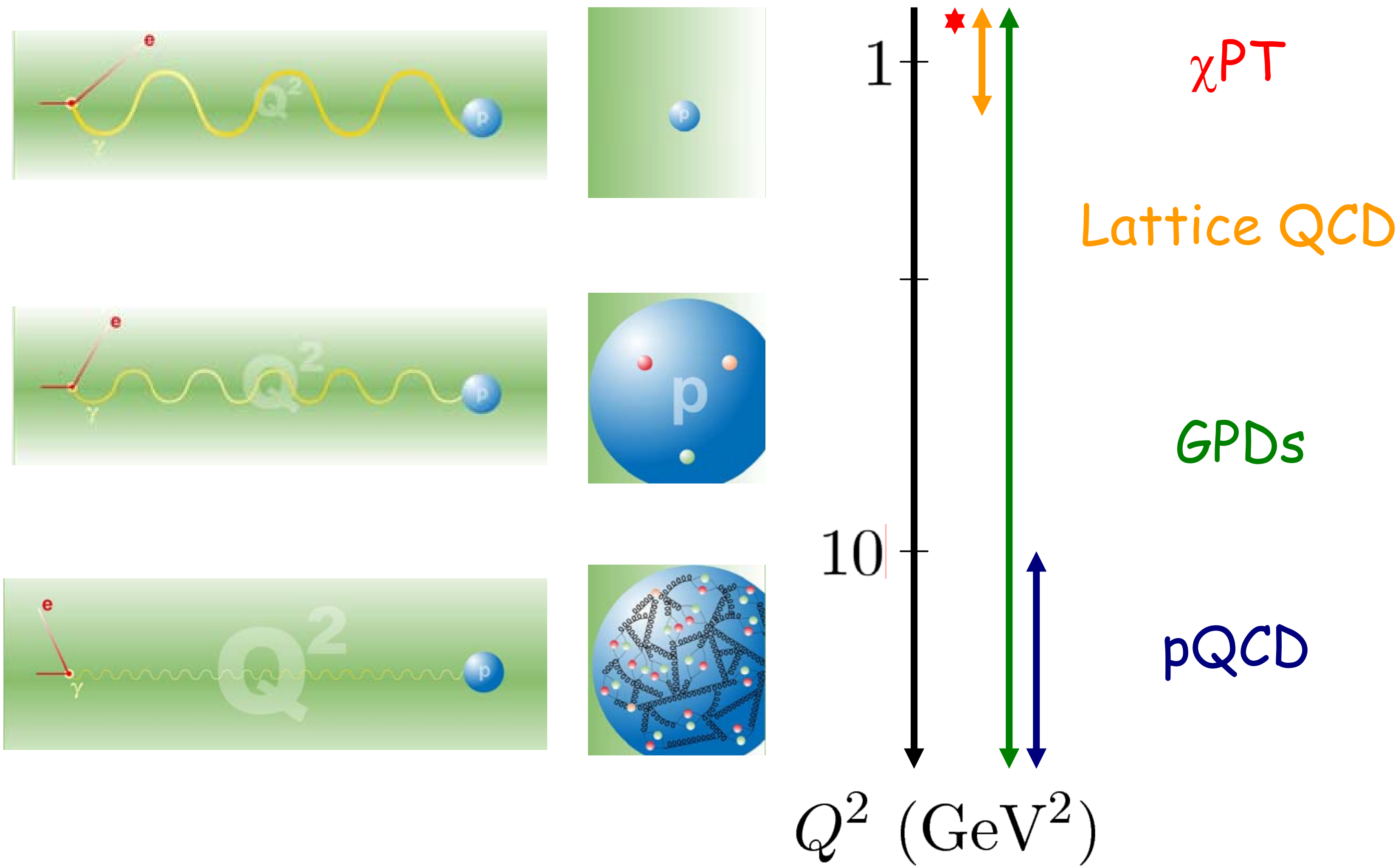




new MIT-Bates (BLAST) data for both p and n at low Q^2

new JLab/HallA double pol. exp. (spring 2007) : extension up to $Q^2 \approx 3.5$ GeV^2 completed

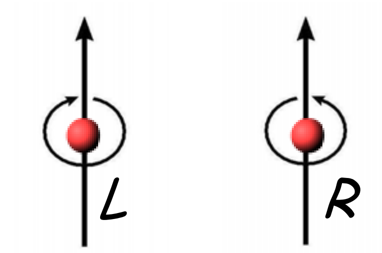
Theoretical Approaches



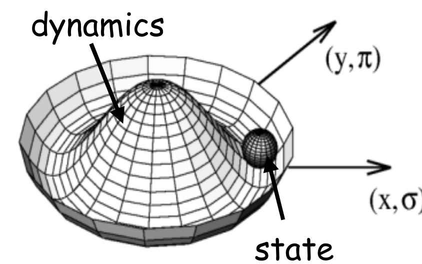
χPT

$$Q^2 \lesssim m_\pi^2 \ll \Lambda_{\chi SB} \sim 1 \text{ GeV}^2$$

Philosophy :



- u, d(, s) quarks almost massless
  - approximate chiral symmetry of QCD

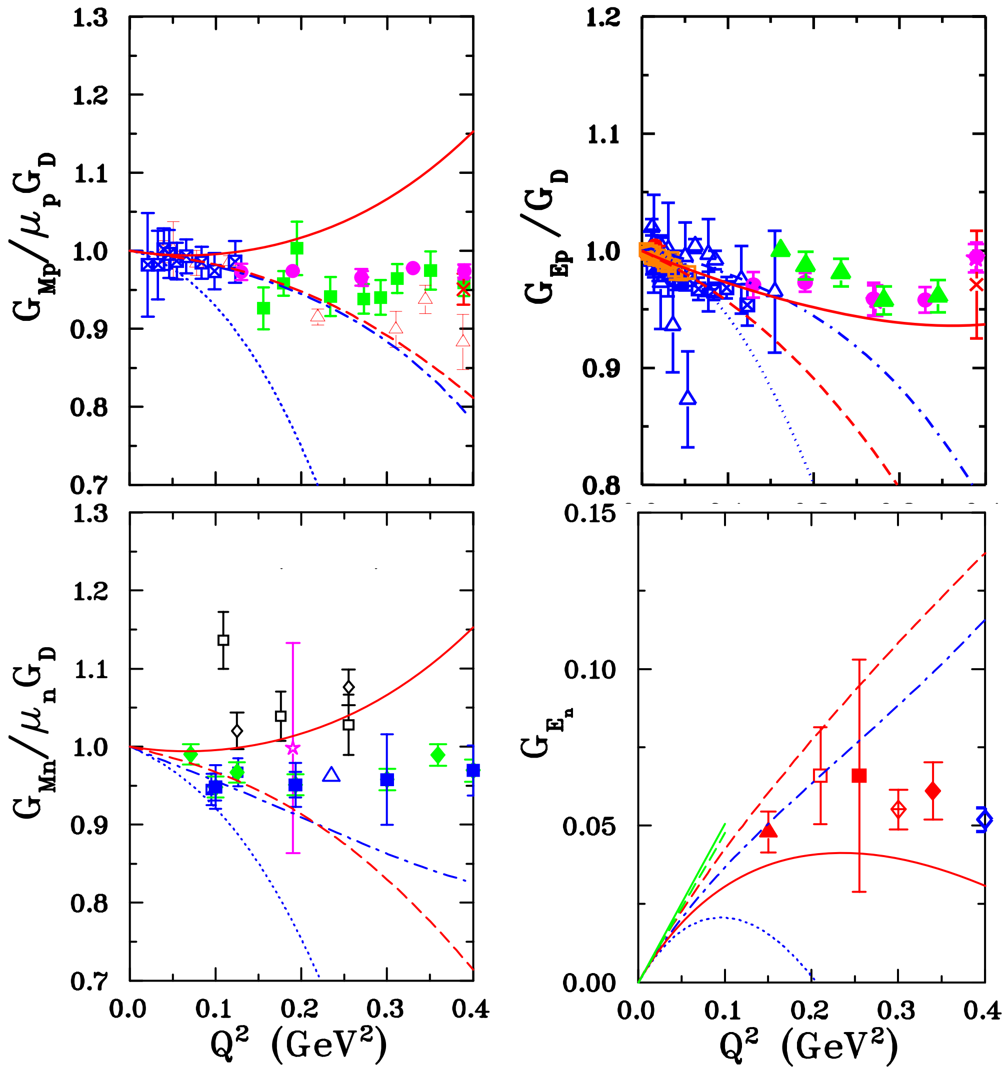


- Low Q^2 : spontaneous symmetry breaking
  - only N, π
- Most general chiral Lagrangian w/ expansion in $|\vec{p}_\pi|$
  - LECs to be fitted
- Explicit Δ, ω, ρ, ϕ , … can be added

talk by V. Pascalutsa

χPT

$$Q^2 \lesssim m_\pi^2 \ll \Lambda_{\chi SB} \sim 1 \text{ GeV}^2$$



[Kubis & Meißner (01)]

(IR scheme)

4th order + VM

3th order + VM

[Schindler *et al.* (05)]

(EOM renorm. scheme)

4th order + VM

4th order, no VM

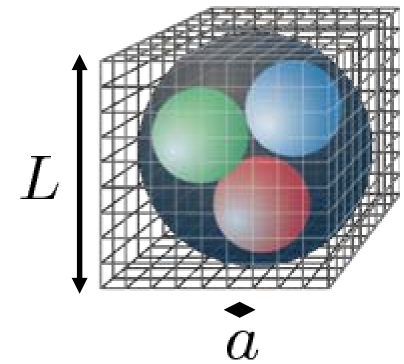
talk by M. Schindler

Lattice QCD

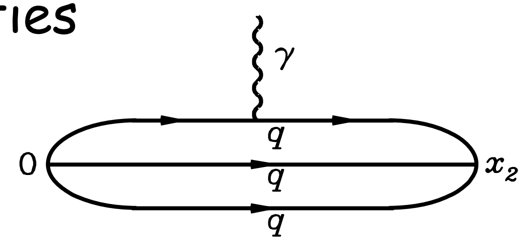
$$\underbrace{Q^2_{min}}_{L} \lesssim Q^2 \lesssim \underbrace{Q^2_{max}}_{a} \sim 2\ \mathrm{GeV}^2$$

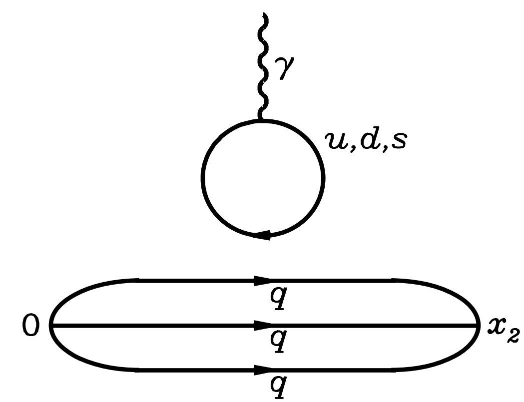
(noise)

Philosophy :



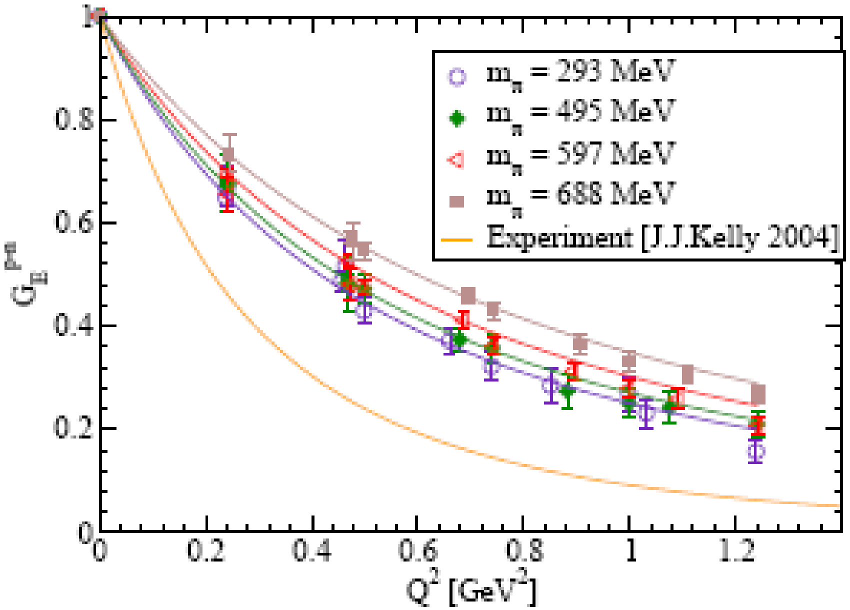
- Discretized Euclidean space-time
  - Path integrals + Monte-Carlo
- Finite-volume effects small : $Lm_\pi \gtrsim 5$
  - Unphysical pion mass → χPT extrapolation
- Often quenched calculations
  - Isovector quantities





Lattice QCD

$$Q^2_{min} \lesssim Q^2 \lesssim Q^2_{max} \sim 2 \text{ GeV}^2$$



LHPC results

(Lattice 2008)

Valence domain-wall fermions on Asqtad staggered sea

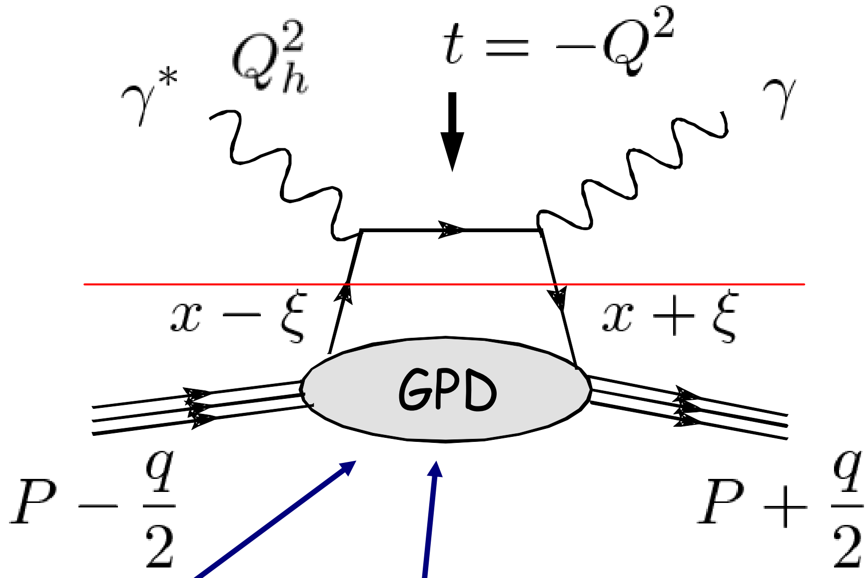
- new $m_\pi = 293$ MeV
- factor 4 reduction in error
- modest m_π dependence

~ factor 2 discrepancy:

- Moderate finite volume effects
- Chiral extrapolation @ « large » Q^2 ?
- Systematic effects (hypercubic symmetry)?

talk by P. Haegler

GPDs



Philosophy :

- DVCS $Q_h^2 \gg Q^2, M^2$
  - Factorization theorem
- Non-perturbative object
  - Vector part

$$\bar{N}(p')\left[\gamma^\mu H^q(x,\xi,Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} E^q(x,\xi,Q^2)\right] n_\mu N(p)$$

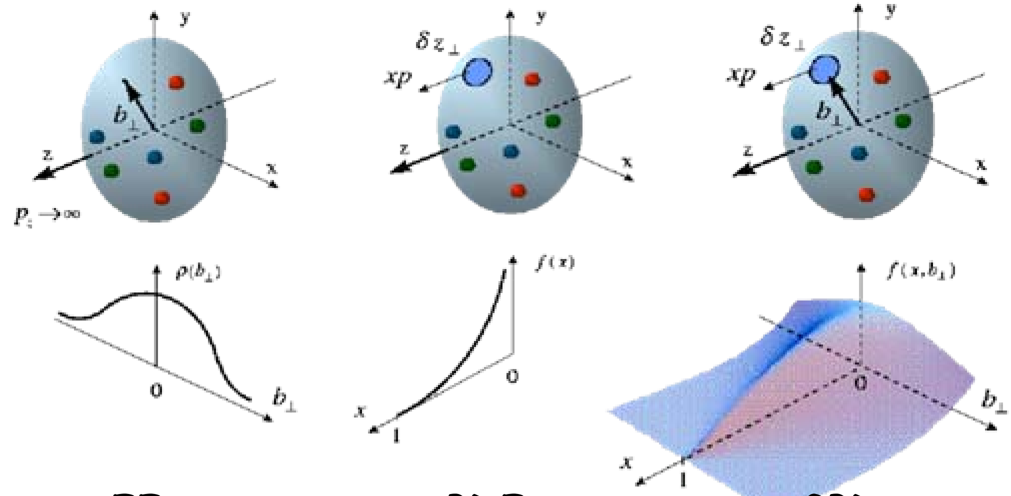
- Model-independent sum rules
  - Constraint on GPDs

$$\int_{-1}^{+1} \mathrm{d}x\, H^q(x,\xi,Q^2) = F_1^q(Q^2), \qquad \int_{-1}^{+1} \mathrm{d}x\, E^q(x,\xi,Q^2) = F_2^q(Q^2)$$

GPDs

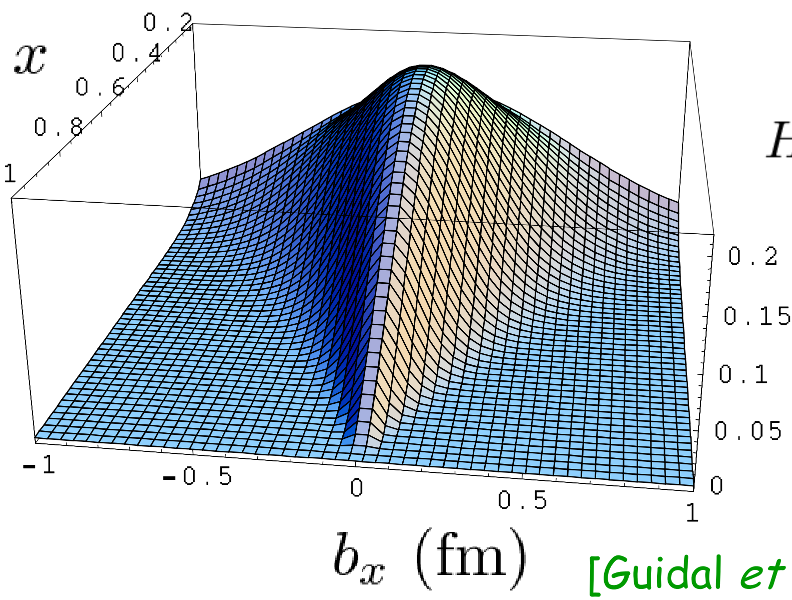
Interpretation :

- Hadron imaging
  - Fourier transform of GPDs

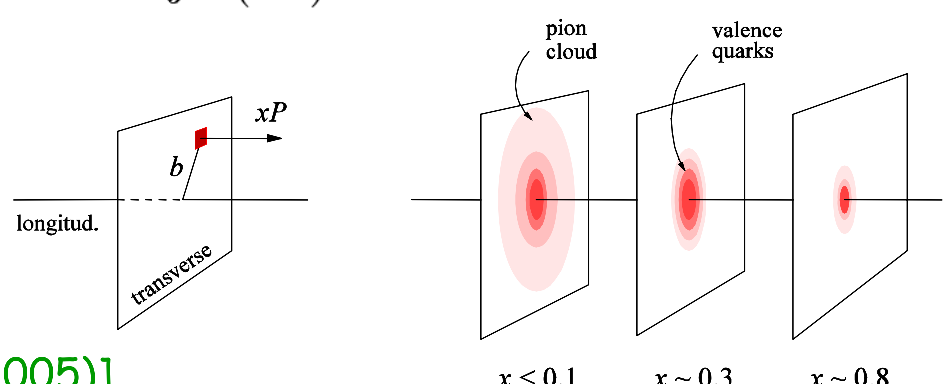


FFs PDFs GPDs

[Belitsky *et al.* (04)]
[Burkardt (01,03)]



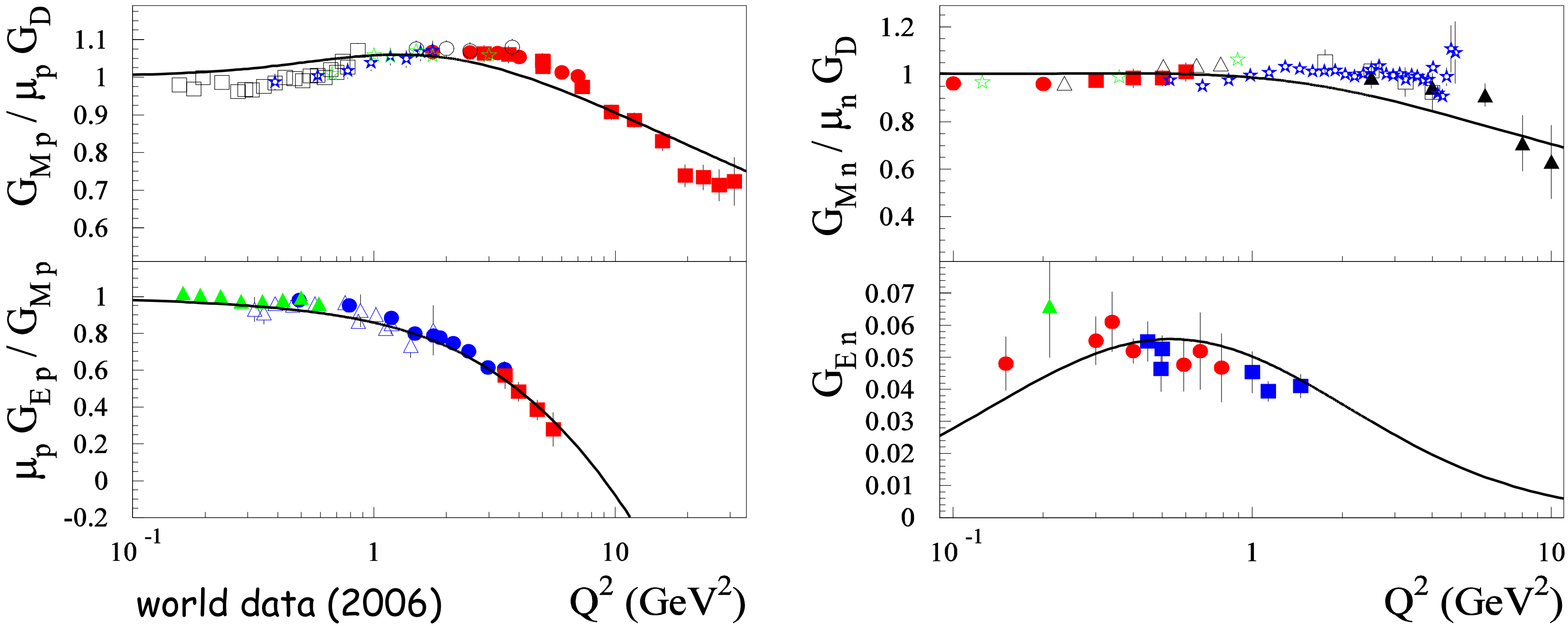
$$H^q(x, \vec{b}_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} H^q(x, \xi = 0, \vec{q}_\perp^{\,2})$$



[Guidal *et al.* (2005)]

GPDs

talk by D. Müller



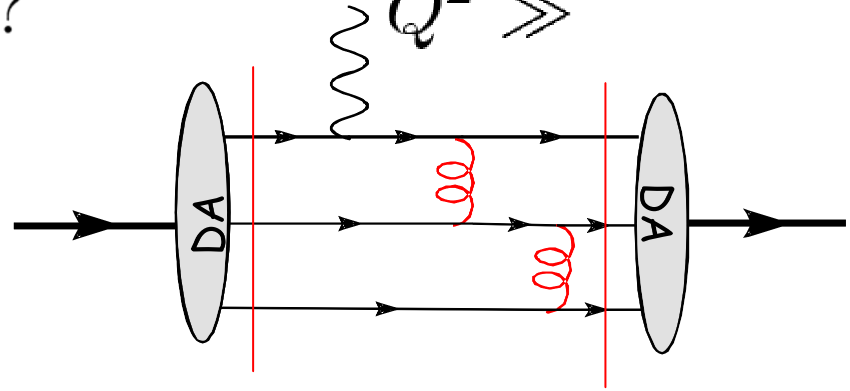
[Guidal *et al.* (05)]
[Diehl *et al.* (05)]

Modified Regge GPD parameterization

3-parameter fit {
- 1 : Regge slope -> proton Dirac (Pauli) radius
- 2, 3 : large x behavior of GPD E^u, E^d -> large Q^2 behavior of F_{2p}, F_{2n}

pQCD

$Q^2 \gtrsim 10 \text{ GeV}^2?$

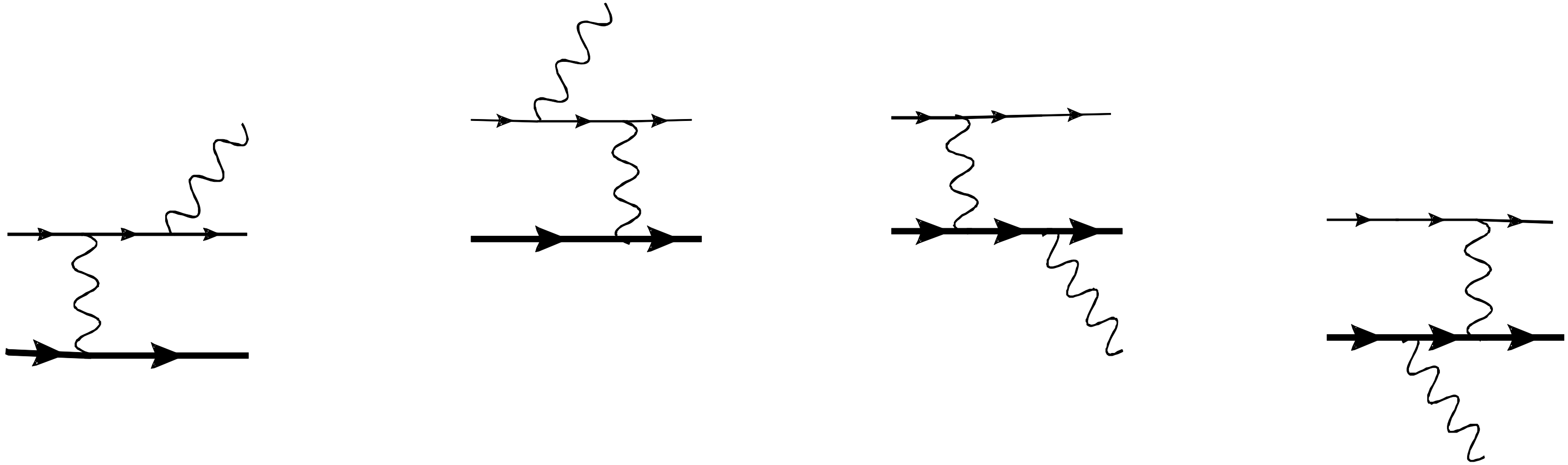


Philosophy :

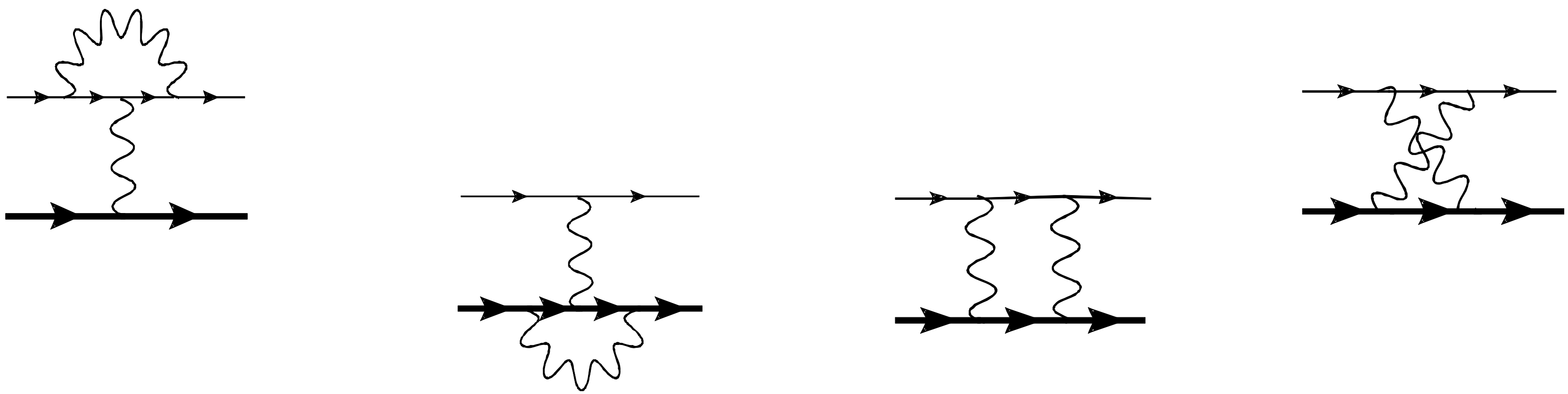
- Very large photon virtuality
  - Photon sees 3 massless (collinear) quarks
- Factorization
  - 2 DAs + 2 hard gluon exchanges
- pQCD predicts a scaling at $Q^2 \gg$

$$F_1 \sim \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{Q^4}, \qquad \frac{F_1}{F_2} \sim \left(\frac{\alpha}{\pi}\right)^2 \frac{\ln^2(Q^2/\Lambda^2)}{Q^2}$$

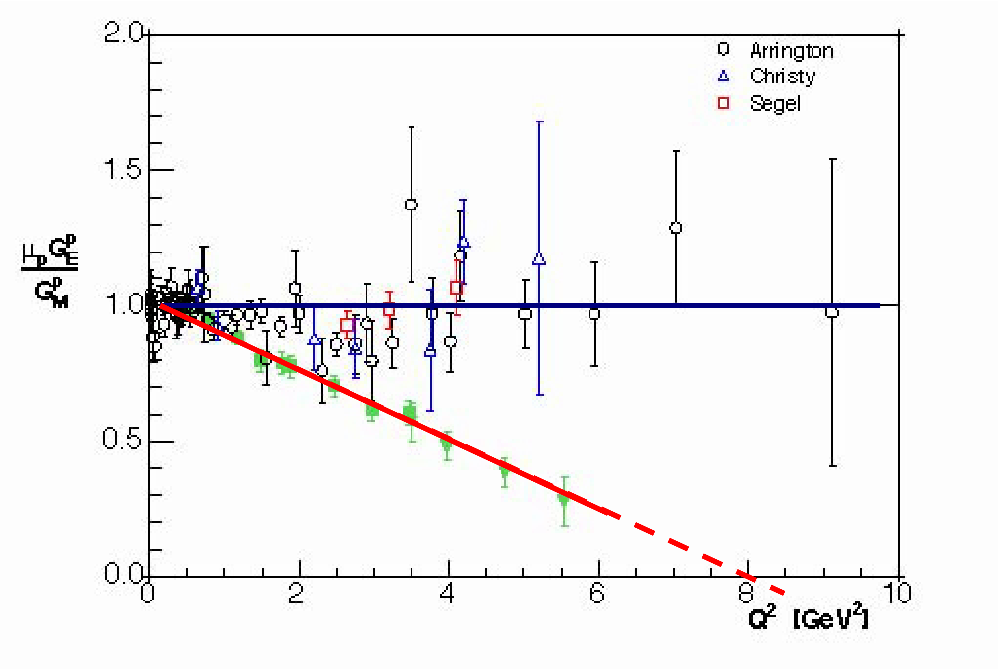
NB: $\left(\frac{\alpha}{\pi}\right)^2 \sim 0.01 \text{ @ } 10 \text{ GeV}^2$



Radiative Corrections



G_E/G_M Extraction methods



Jlab/Hall A
Polarization data

[Jones *et al.* (00)]
[Gayou *et al.* (02)]

$$\vec{e} + p \rightarrow e + \vec{p}$$

$$\frac{P_t}{P_l} \propto \frac{G_E}{G_M}$$

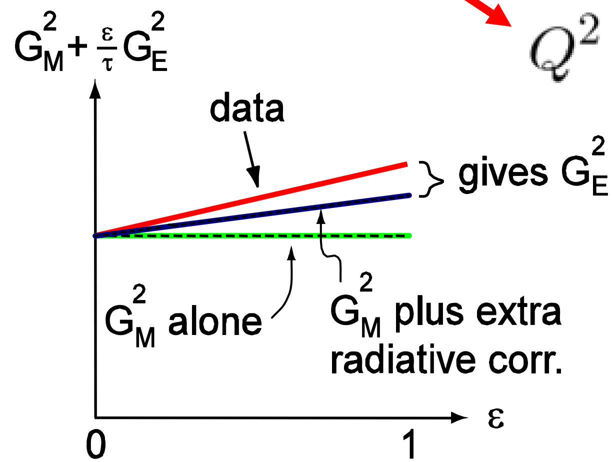
Absolute normalization drops out!

SLAC, Jlab
Rosenbluth data

$$\frac{d\sigma}{d\Omega_{lab}} \propto G_M^2 + \frac{\varepsilon}{\tau} G_E^2$$

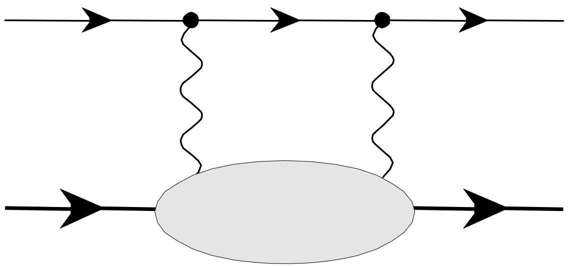
Q^2, θ

Q^2



Speculation :
missing important rad. corr. to Rosenbluth extractions!

Two-γ exchange



$$|\mathcal{M}_\gamma + \mathcal{M}_{2\gamma}|^2 \approx |\mathcal{M}_\gamma|^2 + 2\mathcal{R}\left(\mathcal{M}_\gamma \mathcal{M}_{2\gamma}^*\right)$$

talk by N. Kivel

Rosenbluth

$$\sigma_R = G_M^2 \left(1 + 2\frac{\mathcal{R}\left(\delta\tilde{G}_M\right)}{G_M}\right) + \varepsilon\left\{\frac{1}{\tau}G_E^2\left(1+2\frac{\mathcal{R}\left(\delta\tilde{G}_E\right)}{G_E}\right) + 2G_M^2\left(1+\frac{1}{\tau}\frac{G_E}{G_M}\right)\underbrace{\frac{\nu}{M^2}\frac{\mathcal{R}\left(\tilde{F}_3\right)}{G_M}}_{Y_{2\gamma}}\right\} + \mathcal{O}(e^4)$$

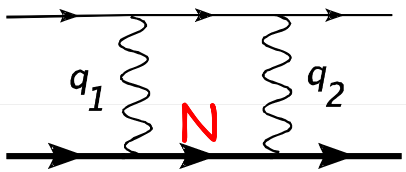
Polarization

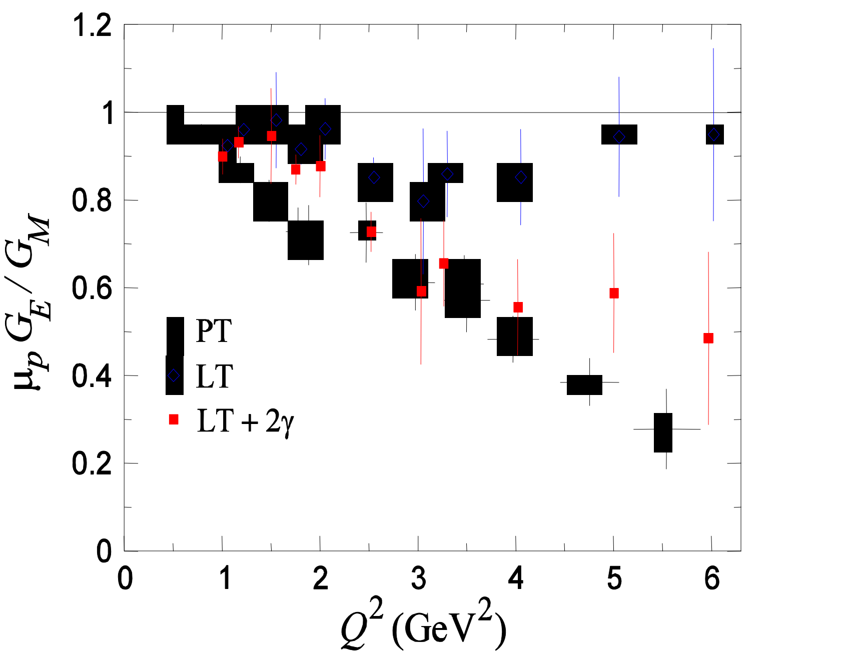
$$\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \left\{\frac{G_E}{G_M}\left(1 - \frac{\mathcal{R}\left(\delta\tilde{G}_M\right)}{G_M}\right) + \frac{\mathcal{R}\left(\delta\tilde{G}_E\right)}{G_M} + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\frac{G_E}{G_M}\right)\underbrace{\frac{\nu}{M^2}\frac{\mathcal{R}\left(\tilde{F}_3\right)}{G_M}}_{Y_{2\gamma}}\right\} + \mathcal{O}(e^4)$$

(green highlight) $\mathcal{O}(\text{few }\%)$

Model Calculations

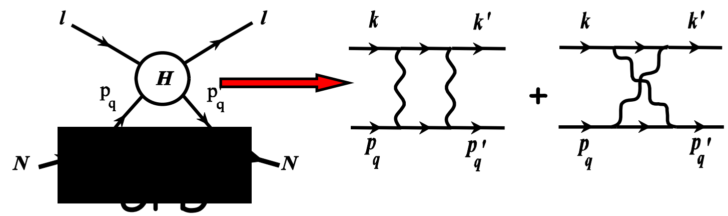
Elastic contribution

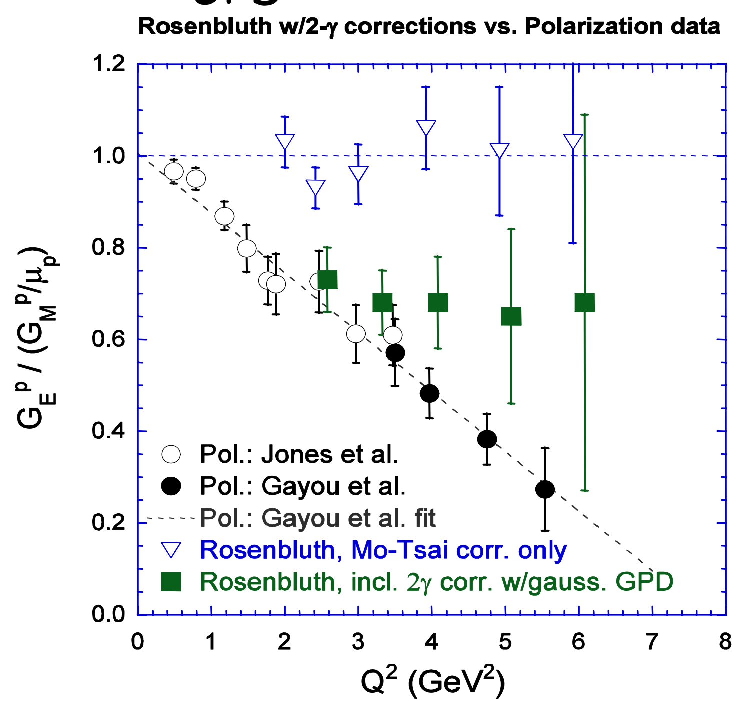




[Blunden *et al.* (03,05)]

Partonic calculation





[Chen *et al.* (03)]

Is two-γ exchange entirely responsible for the discrepancy in the FF extraction?

To be determined experimentally!

Real part of $Y_{2\gamma}$

1) ε-independence of G_{Ep}/G_{Mp} in recoil polarization → Hall C 04-019, completed

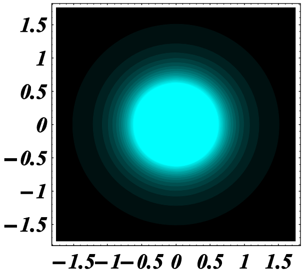
2) cross section difference in e^+ and e^- proton scattering → Hall B 07-005; Olympus/Doris with refurbished BLAST detector

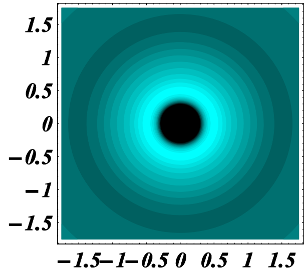
3) non-linearity of Rosenbluth plot → Hall C 05-017; being analyzed

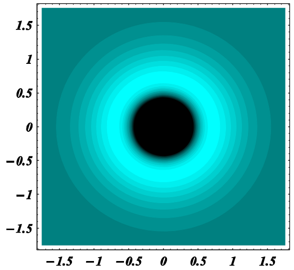
Imaginary part of $Y_{2\gamma}$

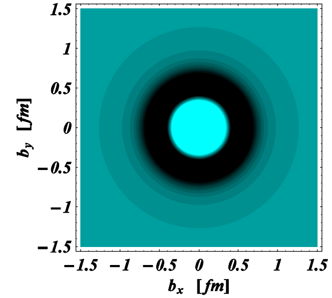
4) from induced out-of-plane polarization → by-product of 04-019/04-108?

5) single-spin target asymmetry → Hall A 05-015 (^3He ↑)

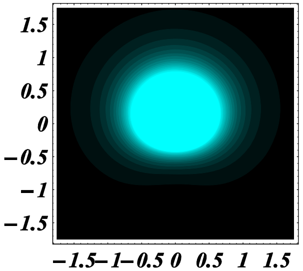


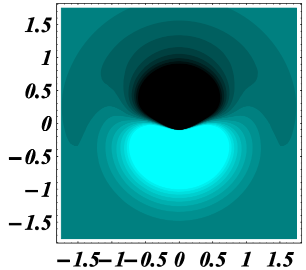


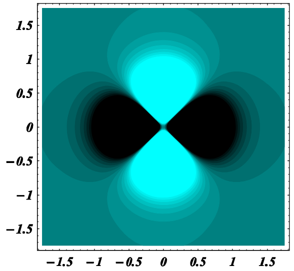


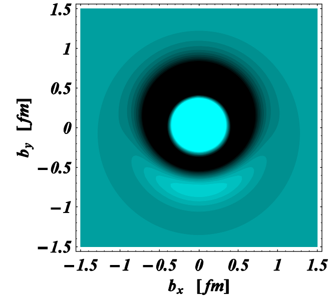


Interpretation of FFs



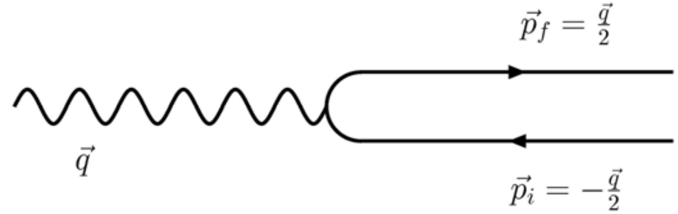






Standard Picture

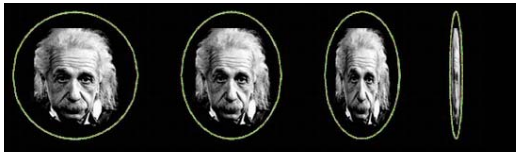
Breit frame



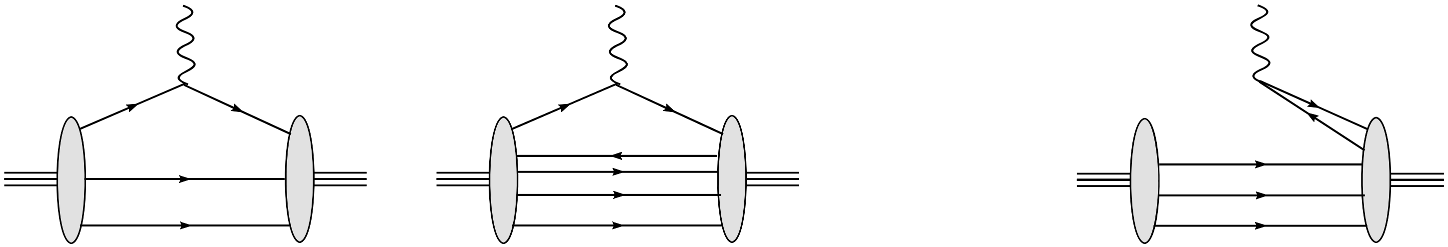
$$\rho(\vec{r}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{r}} G_E(Q^2)$$

BUT

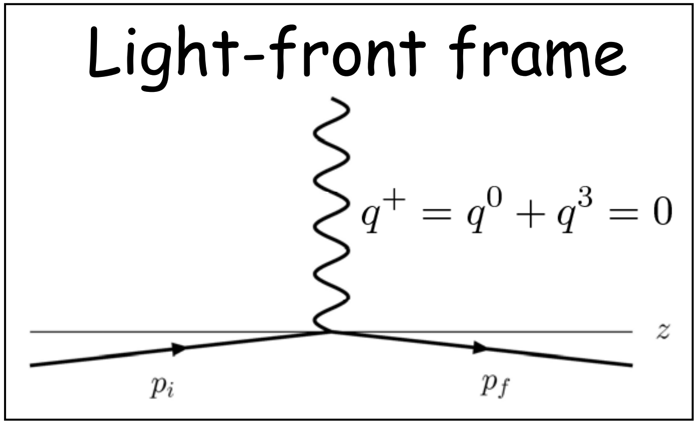
- Lorentz contraction
- Creation/annihilation of pairs



NO probability/charge density interpretation

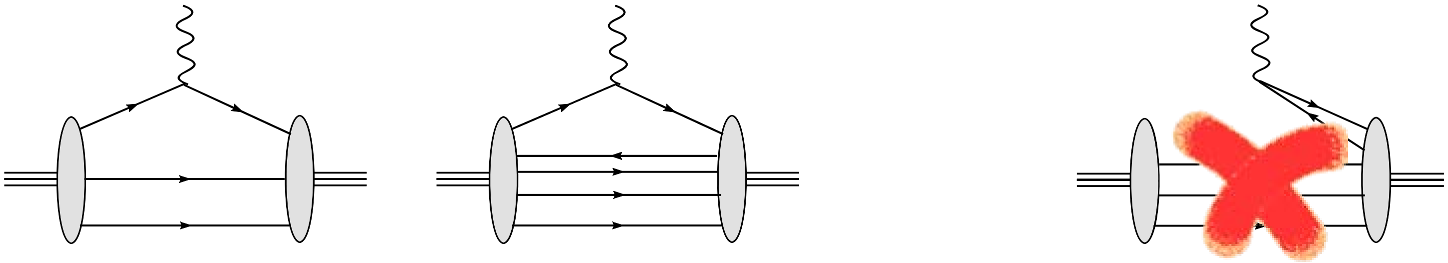


Correct Picture



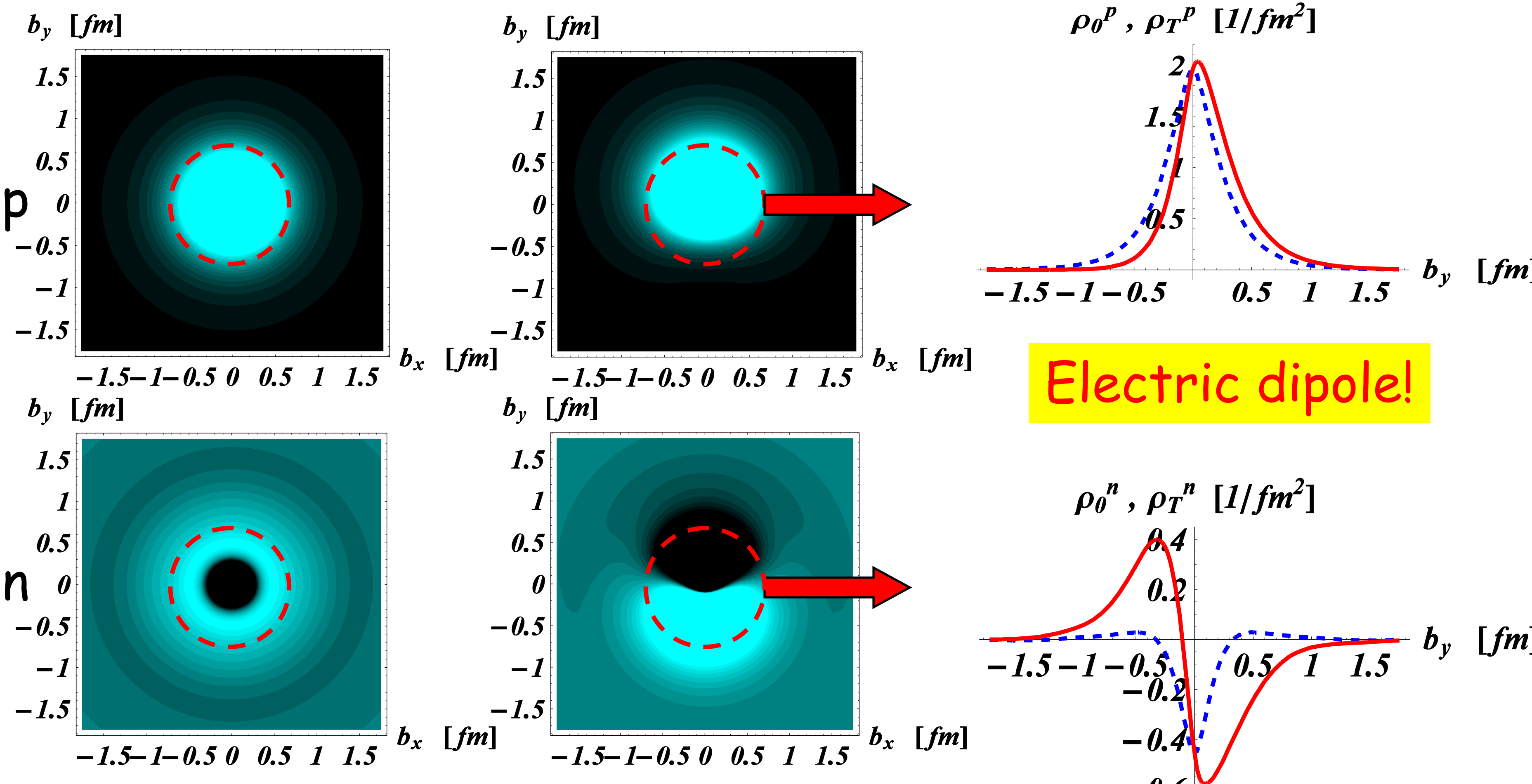
$$\rho_\lambda(\vec{b}) = \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} \frac{e^{-i\vec{q}_\perp \cdot \vec{b}}}{2p^+} \langle p^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | p^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle$$

- Extreme Lorentz contraction → 2D picture
- No creation/annihilation of massive pairs $p^+ > 0$



Transverse charge densities

Long. pol. Transv. pol.



[Miller (07)] [Carlson & Vdh (08)]

Transverse charge densities

Change of basis $|s_\perp = \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}\rangle + |-\frac{1}{2}\rangle\right)$

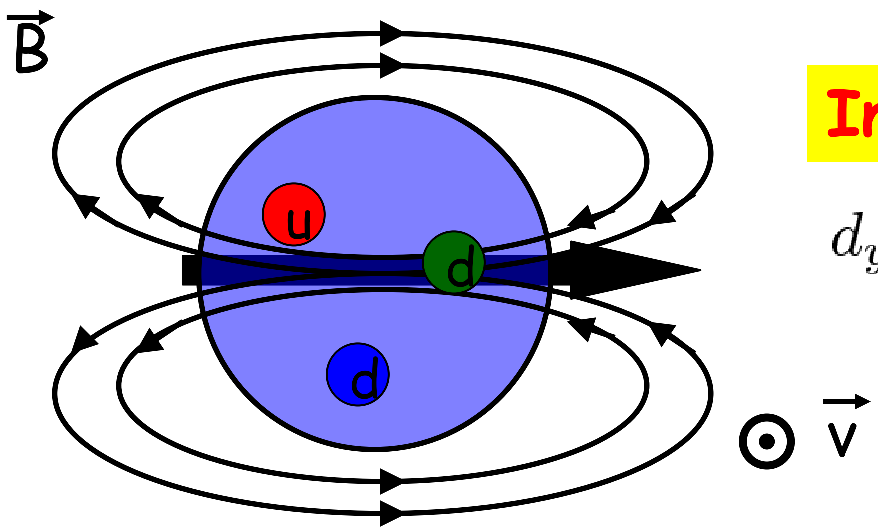
$$\rho^N_{T\,\frac{1}{2}}(\vec{b}) = \int_0^\infty \frac{\mathrm{d}Q}{2\pi}\, Q\, J_0(bQ)\, F_1(Q^2) + \sin\phi_b \int_0^\infty \frac{\mathrm{d}Q}{2\pi}\, \frac{Q^2}{2M}\, J_1(bQ)\, F_2(Q^2)$$

Monopole

Dipole Anomalous!

Interpretation:

$\vec{F} = q\,\vec{v} \times \vec{B}$



Induced EDM

$$d_y = F_2(0)\,\frac{e}{2m}$$

[Carlson & Vdh (08)]

Δ(1232) Resonance

$$Q_{s_\perp}^{\Delta} \equiv e \int \mathrm{d}^2 b \, (b_x^2 - b_y^2) \, \rho_{T_\perp}^{\Delta}(\vec{b})$$

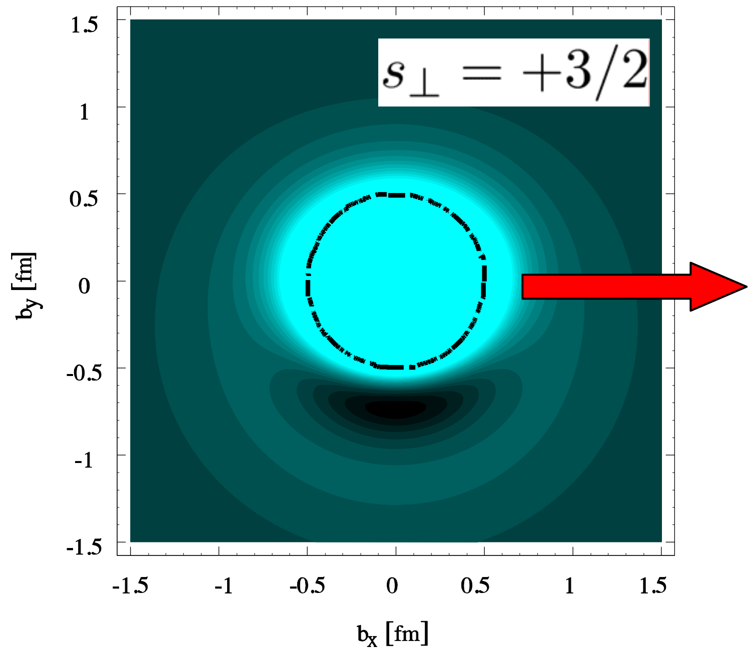
$$Q_{\frac{3}{2}}^{\Delta} = -Q_{\frac{1}{2}}^{\Delta} = \{[G_{M1}(0) - 3e_\Delta] + [G_{E2}(0) + 3e_\Delta]/2\} \frac{e}{M_\Delta^2}$$

Spin-3/2 point particle (SUSY)

$G_{M1}(0) = 3e_\Delta$ and $G_{E2}(0) = -3e_\Delta$

Distortions of transv. charge dens. due to anomalous values of EM moments

→ Internal structure!



Lattice data [Alexandrou *et al.* (08,09)]

Higher Spins

2j+1 circular multipoles!

j	$G_{E0}(0)$ (e)	$G_{M1}(0)$ $(e/2M)$	$G_{E2}(0)$ (e/M^2)	$G_{M3}(0)$ $(e/2M^3)$	$G_{E4}(0)$ (e/M^4)	$G_{M5}(0)$ $(e/2M^5)$
0	1					
$1/2$	1	1				
1	1	2	-1			
$3/2$	1	3	-3	-1		
2	1	4	-6	-4	1	
$\vdots$						
j	C_{2j}^0	C_{2j}^1	$-C_{2j}^2$	$-C_{2j}^3$	C_{2j}^4	C_{2j}^5

Dirac

EW

SM

Supergravity

[Lorcé (09)]

Charge normalization

Universal g=2 factor

$$G_{M1}(0) = 2j$$

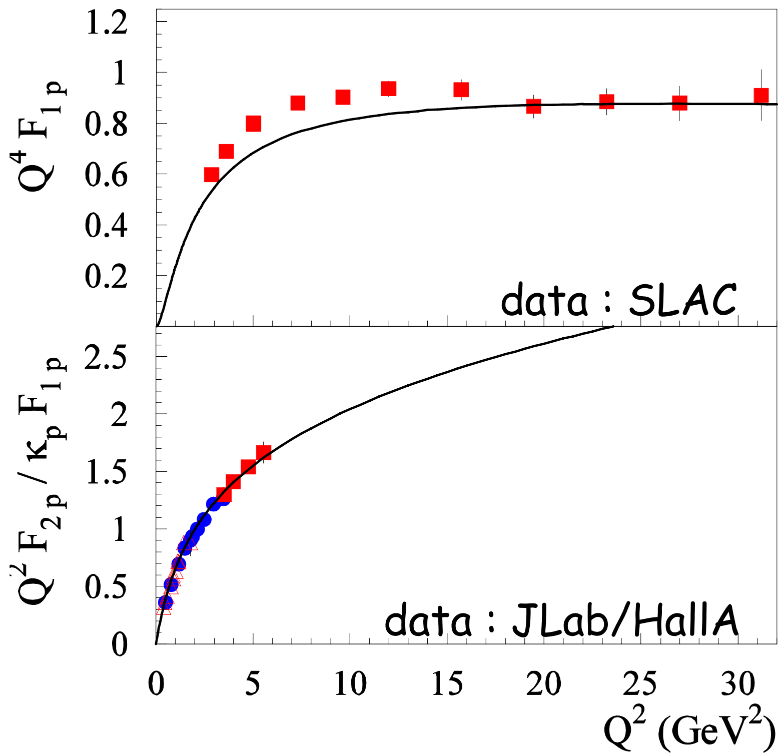
Summary

- Nucleon EM FFs
  - χPT: 4th order +VM
  - Lattice: m_π down to 300 MeV, factor 2 discrepancy
  - GPDs: nucleon imaging constrained by FFs
  - pQCD: scaling at sufficiently large Q^2
- Radiative corrections
  - Discrepancy between Rosenbluth and polarization data -> 2γ?
  - Precision tests: new experiments planned
- Interpretation of FFs
  - Correct picture on the light front
  - Distortions of transverse charge densities due to anomalous moments
  - Natural EM moments form a pseudo-Pascal triangle

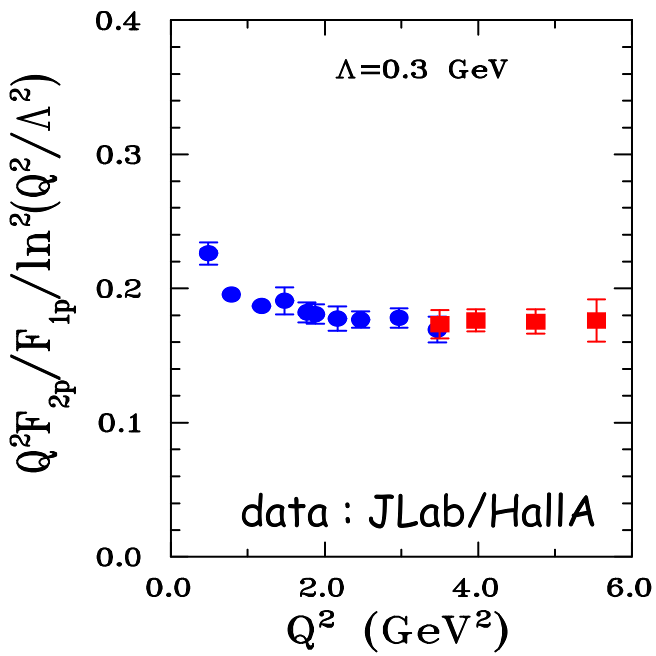
Backup

pQCD $Q^2 \gtrsim 10 \text{ GeV}^2$

Modified Regge GPD model



[Guidal & *al.* (2005)]



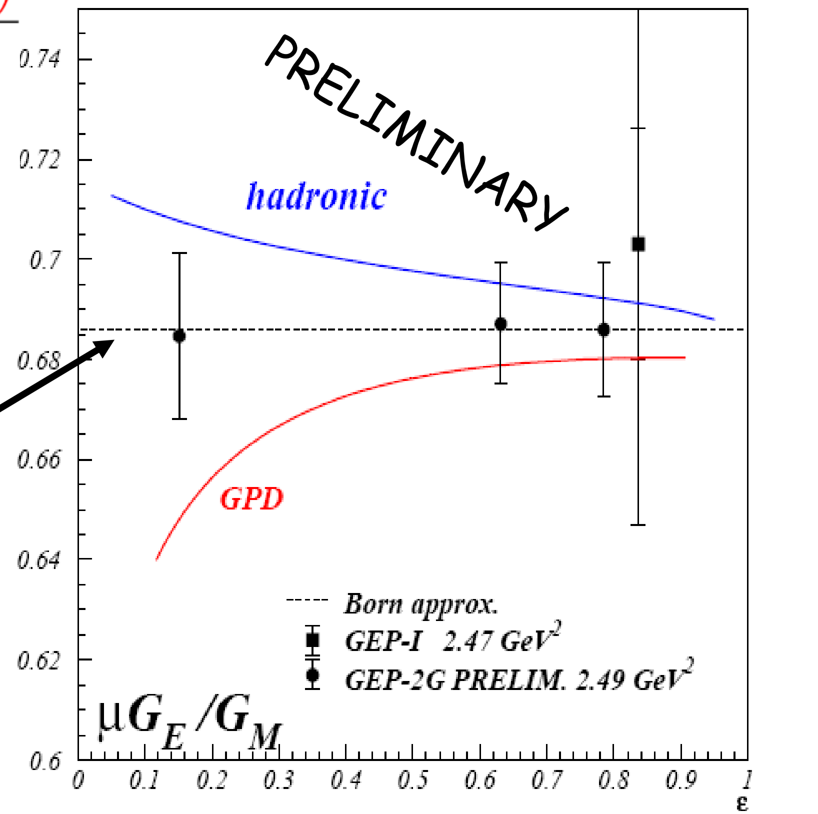
[Belitsky & *al.* (2003)]

Test of ε-dependence of P_t/P_l

new JLab/Hall C data

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\,\varepsilon}{\tau\,(1+\varepsilon)}}\left\{\frac{G_E}{G_M}\left(1-\frac{\mathcal{R}\left(\delta\tilde{G}_M\right)}{G_M}\right)+\frac{\mathcal{R}\left(\delta\tilde{G}_E\right)}{G_M} + \left(1-\frac{2\,\varepsilon}{1+\varepsilon}\,\frac{G_E}{G_M}\right)\frac{\nu}{M^2}\,\frac{\mathcal{R}\left(\tilde{F}_3\right)}{G_M}\right\} + \mathcal{O}(e^4)$$

One-γ result



Preliminary data for Q^2=2.5 GeV^2 show
no ε-dependence of G_{Ep}/G_{Mp} at the 0.01 level